SOLNS: C2 COMPILATION PAPER (2001-2004) {PAPER 1} 75 MARKS

1

(a)	$\log_q 16 = \log_q 2^4$, $\therefore p = 4\log_q 2$ i.e. $\log_q 2 = \frac{p}{4}$	M1, A1	(2)
(b)	$\log_q(8q) = \log_q 8 + \log_q q$	M1	
	$= \cdots + 1$ $\log_q q = 1$	B1	
	$= 3\log_q 2 + \cdots$ $\log_q 8$ in terms of $\log_q 16$ or $\log_q 2$	M1	
	$\therefore \log_q(8q) = \frac{3}{4}p + 1$	A1	(4) 6

2

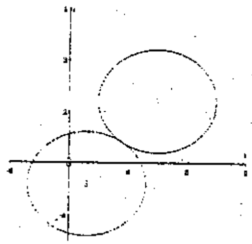
$(2-px)^6 = 2^6 + \binom{6}{1}2^5(-px) + \binom{6}{2}2^4(-px)^2$	Coeff. of x or x^2	M1 $\binom{n}{r}$ okay
$= 64 + 6 \times 2^5(-px); + 15 \times 2^4(-px)^2$	No $\binom{n}{r}$	A1; A1
$15 \times 16p^2 = 135$ $\Rightarrow p^2 = \frac{9}{16}$ or $p = \frac{3}{4}$ (only)		M1, A1
$-6.32p = A$		M1
$\Rightarrow A = -144$		A1 ft (their p (> 0))
		(7 marks)

(ft = follow-through mark)

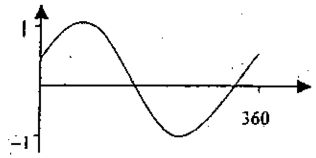
3

(a)	Centre is at (3,-4)	B1	
	radius $= \sqrt{(3^2 + (-4)^2 - -75)} = 10$	M1 A1	(3)

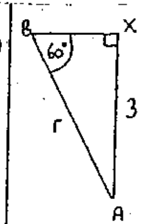
(b)	1st circle	B1	
	2nd circle	B1	
	Circles touching	B1	
	At (9, 4)	B1	(4)



4

(a)		Scales (−1, 1 and 360)	B1
		Shape, position	B1
(b)	(0, 0.5) (150, 0) (330, 0)		B1 B1 B1
(c)	$(x + 30 =)$ 210° or 330°	One of these	B1
	$x = 180°, 300°$	M: Subtract 30, A: Both	M1 A1

5

(a)	 $\sin 60° = \frac{3}{r}$ or $r = 2x$, $4x^2 = x^2 + 3^2$, $x = \sqrt{3}$	M1	
	$r = \frac{6}{\sqrt{3}}$ or $r = 2\sqrt{3}$ or $\sqrt{12}$	A1	(2)
(b)	Area $= \frac{1}{2}r^2\theta^c$ or $\frac{\theta°}{360°} \times \pi r^2$ $=, \frac{1}{6} \times \pi \times 12 = 2\pi$ (cm²)	M1, A1	(2)
(c)	Arc $= r\theta^c$ or $\frac{\theta°}{360°} \times 2\pi r$ $=, \frac{1}{6} \times 2\pi \times 2\sqrt{3}$	M1	
	Perimeter = Arc + 2r $=, \frac{2\sqrt{3}}{3}\pi + 2 \times 2\sqrt{3} = \frac{2\sqrt{3}}{3}(\pi + 6)$ (cm)	M1, A1 c.s.o.	(3) 7

6

(a)	Uses the remainder theorem with $x = \frac{1}{2}$, or long division, and puts remainder = 0	M1
	To obtain $p + 2q = -35$ or any correct equivalent (allow more than 3 terms)	A1
	Uses the remainder theorem with $x = 1$, or long division, and puts remainder = ± 7	M1
	To obtain $p + q = -21$ or any correct equivalent (allow more than 3 terms)	A1
	Solves simultaneous equations to give $p = -7$, and $q = -14$	M1 A1 (6)
(b)	Then $6x^3 - 7x^2 - 14x + 8 = (2x-1)(3x^2 - 2x - 8)$	M1 A1 ft
	So $f(x) = (2x-1)(3x+4)(x-2)$	B1 (3)

7

(a)	$\frac{a}{1-r} = \frac{1200}{1-r} = 960$	M1 A1
	$960(1-r) = 1200$ $\quad r = -\frac{1}{4}$ $\quad$ (*)	A1
(b)	$T_9 = 1200 \times (-0.25)^8$ $\quad$ (or T_{10})	M1
	Difference $= T_9 - T_{10} = 0.0183105\ldots - (-0.0045776\ldots)$	M1
	$= 0.023$ $\quad$ (or -0.023)	A1
(c)	$S_n = \frac{1200\left(1-(-0.25)^n\right)}{1-(-0.25)}$	M1 A1
(d)	Since n is odd, $(-0.25)^n$ is negative,	M1
	so $\quad S_n = 960(1 + 0.25^n)$ $\quad$ (*)	A1

8

(a)	$(x-3)^2 + (y-4)^2 = 18$ OR EQUIVALENT (accept $(3\sqrt{2})^2$)	M1 A1 (2)
(b)	Use $y = x + 3$ to obtain $(x-3)^2 + (x-1)^2 = 18$	M1
	And thus $2x^2 - 8x = 8$	A1
	Solve quadratic, to obtain $x = 2 \pm \sqrt{8}$, $y = 5 \pm \sqrt{8}$	M1, A1, A1 (5)
(c)	Distance $= \sqrt{((2\sqrt{8})^2 + (2\sqrt{8})^2} = 8$	M1A1 cso (2)

9

(a)	$A: y = 1$ $\quad B: y = 4$	B1
(b)	$\frac{dy}{dx} = \frac{2x}{25}$ $\quad = \frac{2}{5}$ where $x = 5$	M1 A1
	Tangent: $\quad y - 1 = \frac{2}{5}(x-5)$ $\quad (5y = 2x - 5)$	M1 A1
(c)	$x = 5y^{\frac{1}{2}}$	B1 B1
(d)	Integrate: $\frac{5y^{3/2}}{3/2}$ $\quad \left(= \frac{10y^{3/2}}{3}\right)$	M1 A1ft
	$[\ ]^4 - [\ ]_1 = \left(\frac{10 \times 4^{3/2}}{3}\right) - \left(\frac{10 \times 1^{3/2}}{3}\right), \quad = \frac{70}{3} \quad (23\frac{1}{3}, 23.3)$	M1 A1, A1
Alternative for (d):	Integrate: $\frac{x^3}{75}$	M1 A1
	Area $= (10 \times 4) - (5 \times 1) - \left(\frac{1000}{75} - \frac{125}{75}\right), \quad = \frac{70}{3} \quad (23\frac{1}{3}, 23.3)$	M1 A1, A1
	In both (d) schemes, final M is scored using candidate's "4" and "1".	

SOLNS:

C2 COMPILATION PAPER (2001-2004) {PAPER 2} 75 MARKS

1

(a)	$p+6+12+q=-\frac{1}{8}p+\frac{6}{4}-6+q$	M1 , M1
	$\therefore \frac{9}{8}p=-22\frac{1}{2}$	M1
	$p=-20$	A1 (4)
(b)	Remainder $= p+q+18 = p+21$ $(=1)$	B1√ ft on p (1)

2

(a)	$a=4,\ b=5$ (both are required)	B1 (1)
(b)	$(x-4)^2+(y-5)^2=25$	M1A1ft (2)
(c)	Finding the distance between centre and (8, 17). $\sqrt{[(8-a)^2+(17-b)^2}$	M1
	Complete method to find PT. i.e. use Pythagoras theorem and subtraction.	M1
	$PT=11.6$	A1 (3)

3

(a)	$4x+9,\ \ +12\sqrt{x}$	B1, B1 (2)
(b)	$\int\left(4x+12x^{\frac{1}{2}}+9\right)dx=2x^2+8x^{\frac{3}{2}}+9x$ (√ dep. on 3 terms)	M1 A1√
(c)	$[\ldots\ldots]_1^2=\left(8+(8\times 2^{\frac{3}{2}})+18\right)-(2+8+9)$	M1
	$=7+16\sqrt{2}$	M1 A1 (5)
		7

4

$(1+px)^n \equiv 1+npx, +\frac{n(n-1)p^2x^2}{2}+\ldots$	B1, B1
Comparing coefficients: $np=-18,\ \ \frac{n(n-1)}{2}=36$	M1, A1
Solving $n(n-1)=72$ to give $n=9$; $p=-2$	M1 A1; A1 ft (7)
	(7 marks)

5

(a)		$\theta=-15°,\ \theta=345°$ One of these...	B1
	$\theta+75=360-$"60"		M1
	$\underline{\theta=225,\ 345}$		A1 (3)
(b)	$(2\theta)=44.4$		B1
	$(2\theta)=\quad 135.6$	One more soln.	B1√
	$(2\theta)=404.4,\quad 495.6$	Other 2 in range	B1√
	$\underline{\theta=\ 22.2,\ 67.8,\ 202.2,\ 247.8}$	$(\div 2)$	M1 A1 (5)
			8

6

(a)	$\log_2(16x) = \log_2 16 + \log_2 x$	M1
	$= \underline{\underline{4 + a}}$	A1 c.a.o (2)
(b)	$\log_2\left(\frac{x^4}{2}\right) = \log_2 x^4 - \log_2 2$	M1
	$= 4\log_2 x - \log_2 2$	M1
	$= \underline{\underline{4a-1}}$ (accept $\underline{\underline{4\log_2 x - 1}}$)	A1 (3)
(c)	$\frac{1}{2} = 4 + a - (4a-1)$	M1
	$a = \frac{3}{2}$	A1
	$\log_2 x = \frac{3}{2} \Rightarrow x = 2^{\frac{3}{2}}$	M1
	$\underline{\underline{x = \sqrt{8} \text{ or } 2\sqrt{2}}}$ or $\underline{\underline{\sqrt{2^3} \text{ or } (\sqrt{2})^3}}$	A1 √ (4)
		(9)

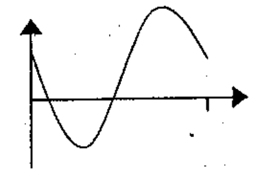
Notes

(a)	M1	Correct use of $\log(ab) = \log a + \log b$
(b)	M1	Correct use of $\log\left(\frac{a}{b}\right) = \cdots$
	M1	Use of $\log x^n = n\log x$
c)	M1	Use their (a)&(b) to form equ in a
	M1	Out of logs: $x = 2^a$
	A1 √	Must write x in surd form, follow through their rational a.

8

(a)	$\sin 2\theta \div \cos 2\theta = \tan 2\theta$, $\tan 2\theta = 0.5$ *	M1 (1)
(b).	$\tan 2\theta = 0.5$, $2\theta = 26.6°$	B1
	$2\theta = 206.6$, One more soln.	B1 √
	386.6, 566.6 Other 2 solns in range	B1 √
	$\underline{\theta = 13.3,\ 103.3,\ 193.3,\ 283.3}$ (M: dividing by 2)	M1 A1 (5)
		(6)

7

(a)	 Shape	B1
	Position	B1 (2)
(b)	$\left(0, \frac{1}{\sqrt{2}}\right)$, $\left(\frac{\pi}{4}, 0\right)$, $\left(\frac{5\pi}{4}, 0\right)$	B1 B1 B1 (3)
(c)	$\left(x + \frac{\pi}{4} =\right)$ $\frac{\pi}{3}$	B1
	Other value $\left(2\pi - \frac{\pi}{3} =\right)$ $\frac{5\pi}{3}$	M1
	Subtract $\frac{\pi}{4}$ $x = \frac{\pi}{12}, x = \frac{17\pi}{12}$	M1 A1 (4)
		9

9

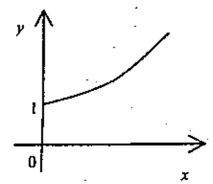
(a)	$100 = 81 + 25 - (2 \times 9 \times 5 \cos BAC)$	M1 A1
	$\cos BAC = \frac{81 + 25 - 100}{90}$ $\left(= \frac{1}{15}\right)$, $BAC = \underline{1.504 \text{ radians}}$. *	A1 (3)
(b)	$\frac{1}{2}r^2\theta = \frac{1}{2} \times 9 \times 1.504 = \underline{6.768 \text{ cm}^2}$ (6.77)	M1 A1 (2)
(c)	Area of triangle $= \frac{1}{2} \times 45 \times \sin 1.504$ ($= 22.450 \text{ cm}^2$)	M1 A1
	Shaded area $= 22.450 - 6.768 = \underline{15.682 \text{ cm}^2}$ (15.68, 15.7)	A1 (3)
(d)	Arc length $= r\theta = 3 \times 1.504$ ($= 4.512$ cm)	M1 A1
	Perimeter $= 10 + 6 + 2 + 4.512 = \underline{22.512 \text{ cm}}$ (22.51, 22.5)	M1 A1√ (4)
		(12)

SOLNS C2 COMPILATION PAPER (2001-2004) PAPER 3 — 69 MARKS

1

(a)	*Complete* attempt at remainder theorem, or long division Either $f(3) = 27 + 9a + 3b - 10 = 14$, Or complete attempt at long division by $(x-3)$ leading to equation. Either $f(-1) = -1 + a - b - 10 = -18$ or long division by $(x+1)$ leading to equation.	M1	
	Equation equivalent to $9a + 3b = -3$ $(3a + b = -1)$	A1	
	Equation equivalent to $a - b = -7$	A1	
	Solve two equations to get $a = -2$, $b = 5$	M1, A1	(5)
(b)	Either $f(2) = 8 - 8 + 10 - 10 = 0$, or complete division with no remainder. $\therefore (x-2)$ is a factor. Or $f(x) = (x-2)(x^2+5)$	M1, A1 cso (M1 A1)	(2)

2

(a)		Shape	B1	
		domain, intercept	B1	2
(b)	$£800 \times 1.04^{10} \approx £1184$	cao	M1 A1	2
(c)	$1.04^x = 2$		M1	
	$x = \dfrac{\ln 2}{\ln 1.04} \approx 18$ (years)		M1 A1	3 7

3

(a)	Attempting to get to $a^6 =$ from $800 = \dfrac{2000a^6}{4+a^6}$	M1
	$a^6 = \dfrac{3200}{1200}$	A1
	$a = \left(\dfrac{3200}{1200}\right)^{\frac{1}{6}} \rightarrow 1.1776$ (4 d.p.)	M1 A1 cao (4
(b)	Substituting $P = 1800$ into formula with a^t as unknown	M1
	$a^t = 36 \rightarrow$, $t = 22$	A1, M1
	Number of years needed for P from 800 to 1800 $= 16$ years	A1 f.t. (4
(c)	$P = \dfrac{2000}{1+4a^{-t}}$, $4a^{-t} \to 0$ as $t \to \infty$ so $P \to 2000$ but does not exceed it	B1 (9) (1

4

(a)	$2x^{3/2} - 3x^{-3/2} = 0$		M1	
	$\Rightarrow$ $x^3 = 3/2$	$x = \sqrt[3]{\alpha}$	M1	
	$x = \sqrt[3]{3/2} = 1.1447\ldots$ $= \underline{1.14}$ 3sf		A1 c.a.o.	(3)
(b)	$f(x) = 4x^3 + 9x^{-3} - 12 + 5$	$A = 4$	B1	
	$= 4x^3 + \frac{9}{x^3} - 7$	$B = 9$, $C = -7$	B1, B1	(3)
(c)	$\int_1^2 f(x)\,dx = \left[x^4 - \frac{9}{2}x^{-2} - 7x\right]_1^2$	$x^n \to x^{n+1}$	M1 A2 $\int$ ($\int$ their A, B, C) (-1 eeoo)	
	$= \left(2^4 - \frac{9}{2}\cdot 2^{-2} - 14\right) - \left(1 - \frac{9}{2} - 7\right)$	[ ] - [ ]	M1 (use of limits)	
	$= \underline{11\frac{3}{8}}$ or $\underline{11.375}$ or $\underline{\text{AWRT } 11.4}$		A1	(5) (11)

5

(a)	$BM = \sqrt{(7^2 + 24^2)} = 25$ (*)	B1
(b)	$\tan \alpha = \frac{7}{24}$ or equiv. and $\angle BMC = 2\alpha$, or cosine rule	M1 A1
	$\angle BMC = 0.568$ radians (*)	A1
(c)	ΔABM: $\frac{1}{2}(14 \times 24)$ (= 168 mm^2) (or other appropriate Δ)	B1
	Sector: $\frac{1}{2}(25^2 \times 0.568)$	M1 A1
	Total: "168 + 168 + 177.5" = 513 mm^2 (or 514, or 510)	M1 A1
(d)	Volume = "513" × 85 mm^3 (M requires unit conversion)	M1
	= 44 cm^3	A1

6

(a)	$5 + 2x - x^2 = 2$ OR $x^2 - 2x - 3 = 0$	M1
	$(x-3)(x+1) = 0$ $x = -1$, $x = 3$ S.C. One correct answer: B1	M1 A1 (3)
(b)	$\int(5 + 2x - x^2)\,dx = [5x + x^2 - \frac{1}{3}x^3]$	M1 A1
	Using limits: $(15 + 9 - 9) - (-5 + 1 + \frac{1}{3})$ $(= 18\frac{2}{3})$	M1 A1
	Shaded area $= 18\frac{2}{3} - 8 = 10\frac{2}{3}$.	M1 A1 (6) (9)

7

(a)	$\theta - 10 = 15$ $\theta = 25$ ($\cos(\theta - 10) = \cos\theta - \cos 10$, etc, is B0)	B1
	$\theta - 10 = 345$ $\theta = 355$ M: Using 360 – "15" (can be implied) Stating $\theta = 345$ scores M1 A0	M1 A1 (3)
	(Other methods: M1 for <u>complete</u> method, A1 for 25 and A1 for 355)	
(b)	$2\theta = 21.8\ldots$ (α) (At least 1 d.p.) (Could be implied by a correct θ).	B1
	$2\theta = \alpha + 180$ or $2\theta = \alpha + 360$ or $2\theta = \alpha + 540$ (One more solution)	M1
	$\theta = 10.9,\ 100.9,\ 190.9,\ 280.9$ (M1: divide by 2) (A1ft: 2 correct, ft their α) (A1: all 4 correct cao, at least 1 d.p.)	M1 A1ft A1 (5)
(c)	$2\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) = 3$, $2\sin^2\theta = 3\cos\theta$	M1, A1
	$2(1 - \cos^2\theta) = 3\cos\theta$	M1
	$2\cos^2\theta + 3\cos\theta - 2 = 0$	
	$(2\cos\theta - 1)(\cos\theta + 2) = 0$ $\cos\theta = \frac{1}{2}$ (M: solve 3 term quadratic up to $\cos\theta = \ldots$ or $x = \ldots$)	M1 A1
	$\theta = 60$, $\theta = 300$	A1 (6)
		(14)

SOLNS: C2 COMPILATION PAPER (2001-2004) PAPER 4 72 MARKS

1

(a)	$f(-3) = -27 - 27 + 30 + 24 = 0 \Rightarrow (x+3)$ is factor. M: $f(\pm 3)$	M1 A1	2
(b)	$(x+3)(x^2 - 6x + 8)$	M1 A1	
	$(x+3)(x-2)(x-4)$	M1 A1	4

2

(a)	Using $f(\pm 2) = 3$	M1	
	Showing that p = 6 ✱, with no wrong working seen.	A1	(2)
	S.C. If p = 6 used and the remainder is shown to be 3 award B1		
(b)	Attempt to find quotient when dividing $(n+2)$ into $f(n)$ or attempting to equate coefficients.	M1	
	Quotient = $n^2 + 4n + 3$, or finding either $q = 1$ or $r = 3$	A1	
	Finding both $q = 1$ and $r = 3$	A1	(3)
(c)	The product of three consecutive numbers must be divisible by 3	M1	
	Complete argument	A1	(2)

3

$2\sin^2\theta - 2\sin\theta = 1 - \sin^2\theta$	M1
$3\sin^2\theta - 2\sin\theta - 1 = 0$	A1
$(3\sin\theta + 1)(\sin\theta - 1) = 0$ (or attempt by formula)	M1 A1ft
$\sin\theta = -\frac{1}{3}$ $\sin\theta = 1$	A1 (5)
$\theta = -19.5°, -160.5°, \quad 90°$	A1 1ft A1 (3)
Final 3 marks: subtract 1 for each extra soln in range. Ignore extra solutions outside range.	
Special case, if the 2nd M mark has not been earned: Noting that $\sin\theta = 1$ (B1) so $\theta = 90°$ (B1)	8

4

(a)	$1 + nax, + \frac{n(n-1)}{2}(ax)^2 + \frac{n(n-1)(n-2)}{6}(ax)^3 + \ldots$	accept 2!, 3!	B1, B1	2
(b)	$na = 8, \quad \frac{n(n-1)}{2}a^2 = 30$	both	M1	
	$\frac{n(n-1)}{2}\cdot\frac{64}{n^2} = 30, \quad \frac{\frac{8}{a}\left(\frac{8}{a}-1\right)a^2}{2} = 30$	either	M1	
	$n = 16, \quad a = \frac{1}{2}$		A1, A1	4
(c)	$\frac{16.15.14}{6}\cdot\left(\frac{1}{2}\right)^3 = 70$		M1 A1	2 8

5

(a) $2\log x = \log x^2$	B1
Combine logs, e.g. $\log_2\left(\frac{y}{x^2}\right) = 3$	M1
$\frac{y}{x^2} = 2^3$, $y = 8x^2$ (*)	A1 (3)
(b) $14x - 3 = 8x^2$	M1
$(4x-1)(2x-3) = 0$ Roots $\frac{1}{4}$ and $\frac{3}{2}$ $x = \ldots$	M1 A1 (3)
(c) $\log_2 \alpha = \log_2\left(\frac{1}{4}\right) = \log_2(2^{-2}) = -2$ (*)	B1 (1)
(d) $\log_2 1.5 = k$ $2^k = 1.5$	M1
$k = \frac{\log 1.5}{\log 2} = 0.585$	M1 A1√ (3)

7

(a)	Area of $X = 2d^2 + \frac{1}{2}\pi d^2$, Area of $Y = \frac{1}{2}(4d^2)\theta$	B1, M1 A1
	Equate and divide by d^2: $2 + \frac{1}{2}\pi = 2\theta$, $\theta = 1 + \frac{1}{4}\pi$. *	M1 A1 (5)
(b)	$12 + 3\pi$	B1 B1 (2)
(c)	$4d + r\theta$, $= 12 + 6(1 + \frac{1}{4}\pi)$, $= 18 + \frac{3}{2}\pi$	M1, A1, A1 (3)
(d)	$X: 12 + 3\pi = 21.425$ cm, $Y: 18 + \frac{3}{2}\pi = 22.712$ cm	
	Difference = <u>13 mm</u> (or 12.9mm) or 12·88 mm	M1 A1 (2) 12

6

7.(a)	Solve $\frac{3}{2}x^2 - \frac{1}{4}x^3 = 0$ to find $p = 6$, or verify: $\frac{3}{2}\times 6^2 - \frac{1}{4}\times 6^3 = 0$ (*)	B1	(1)
(b)	$\frac{dy}{dx} = 3x - \frac{3x^2}{4}$	M1 A1	
	$m = -9$, $y - 0 = -9(x-6)$ (Any correct form)	M1 A1	(4)
(c)	$3x - \frac{3x^2}{4} = 0$, $x = 4$	M1, A1ft	(2)
(d)	$\int\left(\frac{3x^2}{2} - \frac{x^3}{4}\right)dx = \frac{x^3}{2} - \frac{x^4}{16}$ (Allow unsimplified versions)	M1 A1	
	$[\ldots\ldots]_0^6 = \frac{6^3}{2} - \frac{6^4}{16} = 27$ M: Need 6 and 0 as limits.	M1 A1	(4)
			(11)

8

(a)	$f(x) = \ldots + \binom{n}{2}\frac{x^2}{k^2} + \binom{n}{3}\frac{x^3}{k^3} \cdots$	Attempt both terms	M1
	$2\times\frac{n(n-1)}{2k^2} = \frac{n(n-1)(n-2)}{6k^3}$	Correct eqn. Nc$\binom{n}{r}$ (factorials o.k.)	M1
	$\Rightarrow 6k = n-2$ or <u>$n = 6k+2$</u> (*)		A1 c.s.o. (3)
(b)	$\frac{n(n-1)(n-2)(n-3)}{4!\,k^4} = \frac{n(n-1)(n-2)(n-3)(n-4)}{5!\,k^5}$, $\Rightarrow 5k = n-4$ (o.e.)		M1, A1
	Solving: $5k = 6k+2-4$, $\Rightarrow$ <u>$k = 2$ and $n = 14$</u> (*)		M1, A1 cso (4)
(c)	$\left(1+\frac{x}{2}\right)^{14} = 1 + 7x + \binom{14}{2}\frac{x^2}{4} + \binom{14}{3}\frac{x^3}{8} + \binom{14}{4}\frac{x^4}{16} + \binom{14}{5}\frac{x^5}{32}\ldots$		M1 (≥3 correct)
	$= \underline{1 + 7x, + \frac{91}{4}x^2 + \frac{91}{2}x^3, + \frac{1001}{16}x^4 + \frac{1001}{16}x^5 \ldots}$		B1, A1, A1 (4)
			11

SOLNS: C2 COMPILATION PAPER (2001-2004) PAPER 5 — 74 MARKS

1

(a) uses $f(1)=9, \Rightarrow a+b=2$ (o.e.)	M1, A1	(2)
(b) uses $f(-2)=0, \Rightarrow -8a+4b=-28$ (o.e.)	M1, A1	
$\therefore \underline{a=3}, \underline{b=-1}$ (Solves to find both values - M1)	M1 A1 cao.	(4)
		(6)

2

(i) Divide: $1+2x^{-1}$	M1 A1	
Differentiate: $6x^2+\frac{1}{2}x^{-\frac{1}{2}}-2x^{-2}$	M1 A2(1,0)	5
(ii) $\frac{x^2}{4}+\frac{x^{-1}}{-1}$	M1 A1 A1	
$[\ ]^4-[\ ]_1 = \left(4-\frac{1}{4}\right)-\left(\frac{1}{4}-1\right)=4\frac{1}{2}$	M1 A1	5

3

(a) $S=a+(a+d)+(a+2d)+\ldots+[a+(n-1)d]$	B1	
$S=[a+(n-1)d]+[a+(n-2)d]+\ldots+a$	M1	
Add: $2S=n[2a+(n-1)d] \quad \Rightarrow \quad S=\frac{1}{2}n[2a+(n-1)d]$	M1 A1	4
(b) $a=54\ 000$ and $n=9$	B1	
$619\ 200=\frac{1}{2}\times 9\times(2\times 54\ 000+8d)$	M1	
$d=3\ 700$	M1 A1	4
(c) $a+(n-1)d=a+10d=54\ 000+10d=£91\ 000$	M1 A1	2
(d) $ar^{n-1}=54\ 000\times 1.06^{10}$ (ft their n)	M1 A1ft	
$=£96\ 700$ or $£97\ 000$	A1	3

4

(a) Adding: $\sin(A+B)+\sin(A-B)=2\sin A\cos B$	M1	
$\left.\begin{array}{l}A+B=X\\A-B=Y\end{array}\right\}\ 2A=X+Y$	M1	
$A=\frac{1}{2}(X+Y),\ B=\frac{1}{2}(X-Y)$	A1	
$\sin X+\sin Y=2\sin\left(\frac{X+Y}{2}\right)\cos\left(\frac{X-Y}{2}\right)$ (*)	A1	(4)
(b) $\sin 4\theta+\sin 2\theta=2\sin 3\theta\cos\theta$	M1	
$\sin 3\theta=0$ (or $\cos\theta=0$)	M1	
$\theta=0°, 60°, 120°, 180°, 240°, 300°$, $90°, 270°$ — 4 correct:	A1	
6 correct:	A1	
8 correct:	A1	(5)

5

(a) $\frac{1}{2}R^2\theta=\frac{49}{2}\theta$ or $\frac{1}{2}r^2\theta=\frac{25}{2}\theta$	B1	
$\frac{1}{2}R^2\theta-\frac{1}{2}r^2\theta=\frac{49}{2}\theta-\frac{25}{2}\theta=12\theta$	M1 A1	(3)
(b) $12\theta=15 \quad \theta=1.25$ (*)	M1 A1	(2)
(c) $R\theta=7\times 1.25$ (or $r\theta=5\times 1.25$)	B1	
$R\theta+r\theta+4=8.75+6.25+4=19$ m	M1 A1	(3)
(d) $\sin 0.625=\frac{x}{5} \quad AD=2x$ $(=5.851$ m$)$	M1	
$6.25-5.85=0.399$ 40 cm (M dep. on previous M)	M1 A1	(3) (11)

6

Scheme	Marks
(a) EITHER expanding	
Using coefficients 1, 5, 10, 10, 5, 1 as necessary	M1
Using powers x^5, $2x^4$, 2^2x^3 etc as necessary	M1
Completing the method	M1
$A = 64$	B1
$B = 160$, $C = 20$	A2,1,0 (6)
OR Substituting values for x	B1
$x = \quad \rightarrow A = 64$	
Forming a first equation in B and C	M1
Forming a second equation in B and C	M1
Solving to complete the method down to either $B =$ or $C =$	M1
$B = 160$, $C = 20$	A2,1,0
(b) Candidates values of A, B, C used to form	
$20x^4 + 160x^2 + 64 = 349$	M1
$4y^2 + 32y - 57 = 0$ [3 term quadratic needed]	A1 f.t.
Solving for y	M1
replacing by x^2 and completion to obtain all relevant values of x	M1
$\pm\sqrt{\frac{3}{2}}$ or AWRT ± 1.22	A1 cao
	(11) (5)

7

Scheme	Marks
(a) $x(x^2-6x+5)$	M1
$= x(x-1)(x-5)$	M1 A1 (3)
(b) 1 and 5	B1√ (1)
(c) $\frac{dy}{dx} = 3x^2 - 12x + 5$	M1 A1
At $x = 1$. $\frac{dy}{dx} = 3 - 12 + 5 = -4$	A1 (3)
(d) $\int(x^3 - 6x^2 + 5x)dx = \frac{x^4}{4} - \frac{6x^3}{3} + \frac{5x^2}{2}$	M1 A1
$[\ldots\ldots]_0^1 = \frac{1}{4} - 2 + \frac{5}{2} \quad \left(= \frac{3}{4}\right) \quad R$	M1 A1√
Evaluating at 5: $\frac{625}{4} - 250 + \frac{125}{2} \quad \left(= -31\frac{1}{4}\right)$	A1
To find S: $-31\frac{1}{4} - \frac{3}{4} = -32$	M1
Total Area $= 32 + \frac{3}{4} = 32\frac{3}{4}$	A1 (7)
	14

SOLNS: C2 COMPILATION PAPER (2001-2007) PAPER 6 75 MARKS

1

(a) centre is $(5, -3)$	[M1 if sign errors]	M1 A1	(2)
(b) radius is 7	[M1 attempts $\sqrt{g^2+f^2-c}$]	M1 A1	(2)
			4

2

(a) $1 + n(3x), + \frac{n(n-1)}{2!}(3x)^2 + \frac{n(n-1)(n-2)}{3!}(3x)^3$	B1, B1	(2)
(b) $\frac{n(n-1)(n-2)}{6}.27 = 10\frac{n(n-1)}{2}.9$	M1	
$n = 12$	A1	(2)
(c) $\frac{n(n-1)(n-2)(n-3)}{4!}(3x)^4$ Coeff: 40095	M1 A1	(2) 6

3

(a)	$\sin 2\theta \div \cos 2\theta = \tan 2\theta$, $\tan 2\theta = 0.5$	*	M1	(1)
(b)	$\tan 2\theta = 0.5$, $2\theta = 26.6°$		B1	
	$2\theta = 206.6$,	One more soln.	B1 √	
	386.6, 566.6	Other 2 solns in range	B1 √	
	$\underline{\theta = 13.3, 103.3, 193.3, 283.3}$	(M: dividing by 2)	M1 A1	(5) 6

4

(a)	$64 - 16 - 28 + c = 0$ $c = -20$	M1 A1	(2)
(b)	$(x-4)(x^2+3x+5)$ (B1 for $(x-4)$)	B1 M1 A1	(3)
(c)	For x^2+3x+5, $b^2-4ac = -11 < 0$, $\therefore$ No real roots.	M1 A1ft	(2) 7

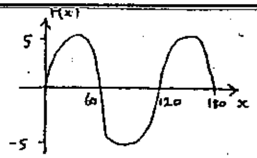
5

(a)	$\frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times 1.5 = 15$	M1 A1
	$r^2 = 20 = \sqrt{(4\times5)}$ $r = 2\sqrt{5}$ (*)	A1
(b)	$r\theta + 2r = 3\sqrt{5} + 4\sqrt{5} = 7\sqrt{5}$ cm (or 15.7, or a.w.r.t 15.65….)	M1 A1
(c)	ΔOAB: $\frac{1}{2}r^2 \sin\theta = 10 \sin 1.5$ (= 9.9749…)	M1
	Segment area = $15 - \Delta OAB = 5.025$ cm^2	M1 A1

6

(a)	$r = 5.12 \div 6.4 = \underline{0.8}$	M1 A1	(2)
(b)	$a = 6.4 \div 0.64 = \underline{10}$ (√: 3 s.f. or better reqd.)	M1 A1√	(2)
(c)	Sum to $\infty = a \div (1-r) = 10 \div (1-0.8) = \underline{50}$	M1 A1	(2)
(d)	$S_{25} = 10(1-0.8^{25}) \div (1-0.8)$ (= 49.8111)	M1 A1√	
	$50 - 49.8111 = \underline{0.189}$ a.w.r.t 0.19	M1 A1	(4) 10

7

(a)		shape B1	[ignore outside (0,180)]
	60, 120, 180 on x-axis	B1	
	5, -5 on y-axis (May be ⇒ by (b))	B1	(3)
(b)	$(30^\circ, 5)$; $(150^\circ, 5)$; $(90^\circ, -5)$ one x-coord	B1	
	all x-co-ords	A1	
	all correct	A1	(3)
(c)	$f(x) = 2.5 \Rightarrow \sin 3x^\circ = \frac{1}{2}$		
	$3x = 30$ (150, 390, 510) one correct value	B1	
	$3x = (\alpha)$ $180-\alpha$, $360+\alpha$, $(540-\alpha)$	M1, M1	
	$x = 10, 50, 130, 170$	A1 (ignore extras out of range)	(4)
			(10)

8

(i) arcsin $0.6 = 36.9^\circ$ (a.w.r.t.) α	B1	
$2x + 50 = 36.87$ $2x = -13.13^\circ + 360^\circ = 346.87^\circ$ M: $(\alpha - 50)$, $+360$	M1, M1	
$2x + 50 = 180 - 36.87$ $2x = 143.13^\circ - 50^\circ = 93.13^\circ$ M: $180 - \alpha$	M1	
$x = 46.6$, 173.4 M: Dividing by 2	M1 A1 A1	7
(ii) (a) $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\frac{BC}{\left(\frac{1}{3}\right)} = \frac{18}{\sin 60^\circ}$	B1, M1	
$BC = 6 \div \frac{\sqrt{3}}{2}$ $BC = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ (" $\frac{12\sqrt{3}}{3}$ " required) (*)	M1 A1	4
(b) $\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{1}{9}$	M1	
$\cos\theta = \sqrt{\frac{8}{9}}$ $\left(= \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}\right)$	A1	2

9

(a)	$x^2 - 2x + 3 = 9 - x$	M1
	$x^2 - x - 6 = 0$ $(x+2)(x-3) = 0$ $x = -2,\ 3$	M1 A1
	$y = 11,\ 6$	M1 A1√
(b)	$\int (x^2 - 2x + 3)\,dx = \frac{x^3}{3} - x^2 + 3x$	M1 A1
	$\left[\frac{x^3}{3} - x^2 + 3x\right]_{-2}^{3} = (9 - 9 + 9) - \left(\frac{-8}{3} - 4 - 6\right)$ $\left(= 21\frac{2}{3}\right)$	M1 A1
	Trapezium: $\frac{1}{2}(11 + 6) \times 5$ $\left(= 42\frac{1}{2}\right)$	B1√
	Area $= 42\frac{1}{2} - 21\frac{2}{3} = 20\frac{5}{6}$	M1 A1
	<u>Alternative:</u> $(9 - x) - (x^2 - 2x + 3) = 6 + x - x^2$	M1 A1
	$\int (6 + x - x^2)\,dx = 6x + \frac{x^2}{2} - \frac{x^3}{3}$	M1 A1√
	$\left[6x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{3} = \left(18 + \frac{9}{2} - 9\right) - \left(-12 + 2 + \frac{8}{3}\right)$, $= 20\frac{5}{6}$	M1 A1, A1

SOLNS: C2 COMPILATION PAPER (2001-2004) PAPER 7 — 73 marks

1

Try to use remainder theorem i.e. evaluate $f(-\frac{1}{2})$ or $f(+\frac{1}{2})$	M1
Uses correct substitution to give $4(-\frac{1}{2})^3+3(-\frac{1}{2})^2-2(-\frac{1}{2})-6=-4\frac{3}{4}$	M1 A1 (3)
Alternative: Uses long division dividing by (2x+1), obtaining $2x^2+\ldots$	M1
Continues as far as remainder, to get $2x^2+\frac{1}{2}x-\frac{5}{4}$ *rem* $-4\frac{3}{4}$	M1, A1 (3)

2

Either		
Obtains centre (0, 6.5)	B1	
Finds radius or diameter by Pythagoras Theorem, to obtain r = 2.5 or $r^2=6.25$	M1, A1	
$x^2+(y-6.5)^2=2.5^2$ or $x^2+y^2-13y+36=0$	B1 ¹ f.t. on $\frac{1}{a}$	(4)
Or		
$\frac{y-8}{x+2}\times\frac{y-5}{x-2}=-1$ Gradients multiplied and put = to -1	B1 M1A1	
$x^2+y^2-13y+36=0$	B1	(4)
Or		
Obtains centre (0, 6.5)	B1	
$x^2+(y-6.5)^2=r^2$ or $x^2+y^2-13y+c=0$	B1	
substitutes either (2, 5) or (-2, 8)	M1	
$x^2+(y-6.5)^2=2.5^2$ or $x^2+y^2-13y+36=0$	A1	(4)

3

(a)	$f(-2)=(-2)^3-(19\times-2)-30$ M: Evaluate $f(-2)$ or $f(2)$	M1	
	$f(-2)=0$, so $(x+2)$ is a factor	A1	(2)
	<u>Alternative:</u> $(x^3-19x-30)\div(x+2)=(x^2+ax+b)$, $a\neq0, b\neq0$ [M1]		
	$=(x^2-2x-15)$, so $(x+2)$ is a factor [A1]		
(b)	$(x^3-19x-30)=(x+2)(x^2-2x-15)$	M1 A1	
	$=(x+2)(x+3)(x-5)$	M1 A1	(4)
			(6)

4

	(a) $4^x=(2^x)^2=u^2$ or $2^{(x+1)}=2.2^x=2u$, $\to u^2-2u-15(=0)$	must be explicit M1, A1 c.s.o. (2)
S.C. 2.32 with no method shown is B1	(b) $u^2-2u-15=(u-5)(u+3)\Rightarrow u=5$ or ... [M1 for $(u\pm5)(u\pm3)$]	M1, A1 ($u=5$) c.s.o.
$\frac{1}{4}$ or $\frac{3}{4}$	$u=5\Rightarrow 2^x=5\Rightarrow x=\frac{\log 5}{\log 2}$, $=2.32$	M1, A1 $\sqrt{}$ (+ve u) (4) Correct use of logs
	[Ignore any other solution]	⑥
ALT	Trial & Improvement on $2^x=5$: M1 for 2 values, one either side, A1 for 2.32	

5

$2\cos^2\theta-\cos\theta-1=1-\cos^2\theta$	M1
$3\cos^2\theta-\cos\theta-2=0$	A1
$(3\cos\theta+2)(\cos\theta-1)=0$ $\cos\theta=-\frac{2}{3}$ or 1	M1 A1
$\theta=0$ $\theta=131.8°$	B1 A1
$\theta=(360-\text{"131.8"})°=228.2°$	M1 A1$\sqrt{}$

6

(a)	$S = a + ar + ar^2 + \dots + ar^{n-1}$	B1	
	$rS = ar + ar^2 + \dots + ar^n$	M1	
	Subtract: $S(1-r) = a(1-r^n)$ $S = \frac{a(1-r^n)}{1-r}$	M1 A1	(4)
(b)	$ar = 3$ $ar^3 = 1.08$	B1 B1	
	Divide: $r^2 = 0.36$ $r = 0.6$	M1 A1	
	$a = 6 \div 1.2 = 5$	A1	(5)
(c)	$S = \frac{5}{1-0.6}$	M1 A1ft	
	$= \underline{12.5}$	A1	(3) (12)

7

(a)	$l = (50-2x)$ $w = (40-2x)$	B1 (either)	
	$V = x(50-2x)(40-2x)$ $(V = xlw)$	M1	
	$V = x(2000 - 80x - 100x + 4x^2) = \underline{4x(x^2 - 45x + 500)}$ (*)	A1 c.s.o.	(3)
(b)	$\underline{x < 20}$ (or in words) (Accept $\leq$)	B1	(1)
(c)	$\frac{dV}{dx} = 12x^2 - 360x + 2000$ (Accept $\div 4$)	M1 $x^n \to x^{n-1}$ A1	
	$\frac{dV}{dx} = 0 \Rightarrow 3x^2 - 90x + 500 = 0 \Rightarrow x = \frac{90 \pm \sqrt{8100 - 6000}}{6}$	M1 $V' = 0$ and attempt to solve	
	$x = (21.6)$ required $x = \underline{7.36 \text{ or } 7.4 \text{ or } 7.362}$	A1 [Ignore 2nd Answer]	(4)
(d)	$V_{max} = 4 \times 7.36(7.36^2 \dots)$, AWRT: $\underline{6564 \text{ or } 6560 \text{ or } 6600}$	M1, A1	(2)
(e)	eg. $V'' = 24x - 360 \Big|_{x=7.36} (= -183\dots) < 0 \therefore$ MAX	M1 Full method	
		A1 Full accuracy (2	(12)

8

(a)	$\frac{1}{2}r^2\theta = \frac{1}{2} \times 6.5^2 \times 0.8 = 16.9$ (a.w.r.t. if changed to degrees)	M1 A1	(2)
(b)	$\sin 0.4 = \frac{x}{6.5}$, $x = 6.5 \sin 0.4$, (where x is half of AB)	M1, A1	
	(n.b. 0.8 rad = 45.8°)		
	$AB = 2x = 5.06$ (a.w.r.t.) (*)	A1	(3)
	Alternative: $AB^2 = 6.5^2 + 6.5^2 - 2 \times 6.5 \times 6.5\cos 0.8$ [M1]		
	$AB = \sqrt{6.5^2 + 6.5^2 - 2 \times 6.5 \times 6.5\cos 0.8}$ [A1]		
	$AB = 5.06$ [A1]		
(c)	$r\theta + 5.06 = (6.5 \times 0.8) + 5.06 = 10.26$ (a.w.r.t) (or 10.3)	M1 A1	(2)
			(7)

9

(a)	$x + 1 = 6x - x^2 - 3$	M1	
	$x^2 - 5x + 4 = 0$ $(x-1)(x-4)$ (or use of formula) $x = \dots$	M1	
	$x = 1$ $x = 4$	A1	
	$y = 2$ $y = 5$	M1 A1	(5)
(b)	$\int (6x - x^2 - 3)\,dx = 3x^2 - \frac{x^3}{3} - 3x$	M1 A1	
	Limits x_A and x_B: $(48 - \frac{64}{3} - 12) - (3 - \frac{1}{3} - 3)$ (= 15)	M1 A1	
	Trapezium: $\frac{1}{2}(2+5) \times 3 = 10.5$	B1ft	
	Area of $R = 15 - 10.5 = \underline{4.5}$	M1 A1	(7) (12)
	Alternative for (b)		
	$(6x - x^2 - 3) - (x + 1) = 5x - x^2 - 4$	M1 A1	
	$\int (5x - x^2 - 4)\,dx = \frac{5x^2}{2} - \frac{x^3}{3} - 4x$	M1 A1ft	
	Limits x_A and x_B: $\left(40 - \frac{64}{3} - 16\right) - \left(\frac{5}{2} - \frac{1}{3} - 4\right)$, $= 4.5$	M1 A1, A1	